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# CRAMER-RAO BOUNDS BASED RSS CALCULATION AND JOINED RSS AND OPTIMAL DOA IN COGNITIVE RADIO NETWORK

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## ABSTRACT

*Contemporary research in communication system much focus towards high transmission rates in mobile systems. In cognitive radio network efficient utilization of bandwidth or available spectrum, create an additional mechanism for primary and secondary user interaction. This paper focus towards algorithms for PU localization based on the received signal strength (RSS) and optimal Direction of arrival (DOA) estimation. Comparative analysis of CRAMER-RAO BOUNDS based RSS calculation and joined RSS and optimal DOA estimator will be performed.*

***Keywords: CRAMER-RAO BOUNDS(CRB),DOA,RSS,PU.***

## I. INTRODUCTION

The ever-increasing demands for higher transmission rates in combination with the emergence of more and more mobile devices and services require an efficient use of the electromagnetic spectrum. Traditionally, dedicated parts of the RF spectrum have been licensed for exclusive use, leading to temporally and spatially unused frequency bands. A lot of recent research has therefore focused on cognitive radio (CR) networks, where cognitive, secondary users (SU) sense the spectrum and dynamically access those bands that are available. However, these secondary networks need to assure that the interference introduced to the primary users (PU) is kept at minimum [1], [2].

When aiming for the usage of temporal spectral holes, it is sufficient to check whether a PU is transmitting or not. As opposed to this, accessing spatial spectrum opportunities with directional antennas as e.g. in [3] requires more detailed information about the primary network. At the transmitting SU, the locations of the PUs must be known in order to direct transmission away from them. In addition, once the PU locations are available, they can be used for routing in the secondary network [4], [5]. The PU locations need to be estimated by observing the PU-transmitted signals as normally no cooperation between the primary and secondary network exists.

The PU localization problem in CR networks is in general different from localization in other applications such as Wireless Sensor Networks (WSN) [6] and Global Positioning System (GPS) [7], due to the following two features. First, a PU does not cooperate or communicate with CRs since they are opportunistic users of the PU spectrum band. Therefore, very limited knowledge about PU signalling, such as transmit power or modulation scheme, is available to CRs. As a result, passive localization techniques should be applied. Second, since CRs need to detect and localize PUs in the whole coverage area at a very low signal-to-noise ratio (SNR), in order to avoid interference to the primary network, the required number of CRs is relatively large and cooperation among CRs is necessary.

Prior research on passive localization can be categorized into three classes based on the types of measurements shared among sensors to obtain location estimates [6]. Received signal- strength (RSS) based algorithms use measured received power from the PU to provide coarse-grained estimates at a low hardware and computational cost. Time-difference-of arrival (TDoA) based algorithms obtain location estimates from time differences among multiple receptions of the transmitted signal. They are not suitable for CR applications since TDoA-based algorithms require perfect synchronization among CRs. Direction-of-arrival (DoA) based algorithms use target DoA estimates observed at different receivers to obtain location estimates.

Algorithms for PU localization based on the received signal strength (RSS) exist in the literature, e.g. [8]. The downside of this approach is that many secondary sensors need to collaborate in order to obtain accurate results since the RSS is heavily influenced by the channel (e.g. shadowing and other uncertainties). Furthermore, the localization works only for a single PU in the observation area because the distinction between multiple PUs is impossible in the RSS domain. Direction of arrival (DOA) estimation, on the other hand, makes it possible to detect multiple transmitters by means of directional antennas as long as at least few SUs cooperate. The DOA can be estimated using antenna array techniques such as MUSIC [9]. For the directional transmission at the SUs, it is actually sufficient to have knowledge about the angles (instead of locations) to all present PUs. However, a problem arises when the signal to noise ratio (SNR) at the SU is too low or the antenna amount in the SU is very small. Then, the

DOA estimation becomes inaccurate [10] or even worse, the PU is not detected at all.

## II.PROPOSED METHODOLOGY

In this paper a PU centric circular region is consider with K CRs. Let us consider the coordinates of PU as $l_p = [x_p, y_p]^T$ and the coordinates of CR $k$as $l_k = [x_k, y_k]^T$ , which is assumed as fusion centre. Let us assume CR is uniformly distributed in circle of radius R and further it can interact with PU, as shown in Fig.1. Following are the calculation of RSS and DOA at CR.

RSS calculation at the *n*th CR is modelled as:

$$\widetilde{\psi_n} \triangleq P_T \frac{c_0 10^{-s_n/10}}{d_n^{\gamma}} \text{ (Watts)} \quad (1)$$

Where $P_T$ the *PU is* transmit power, $c_0$ the (constant) gain at the distance, $d_n = \|l_n - l_p\|$is the distance between the *n*th CR and PU, $\gamma$ is path log exponent and $10^{-s_n/10}$ represents shadowing coefficient.

The RSS is represented as following and expressed in dBm.

$$\phi_n = 10log10\,(1000\ \widetilde{\psi_n})$$

$$\widetilde{\phi_n} = 10log10\,(1000\,P_T\,c_0) - 10\gamma\,\log_{10} dn - s_n \triangleq \overline{\phi_n} - s_n \quad (2)$$

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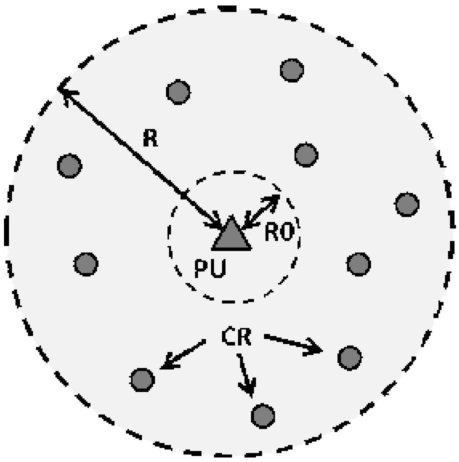


**Figure 1: A circular structure for CR placement**

Denote collection of RSS measurements from all CRs as $\hat{\phi} = [\hat{\phi}_1, \hat{\phi}_2, \dots \hat{\phi}_N]^T$. The conditional distribution of $\hat{\phi}$ (for a given $l_p$)is $\hat{\phi} \sim (\bar{\phi}, \Omega_s)$, where $\bar{\phi} = [\bar{\phi}_1, \bar{\phi}_2, \dots \bar{\phi}_N]^T$, and $\Omega_s$ is covariance matrix of collection of shadowing variables given by $\{\Omega_s\}_{mn} = \sigma_s^2 e^{-\|l_m - l_n\| / X_c}$. Where $X_c$ correlation distance is correlated nodes.

### 2.1. Direction of Arrival (DOA)

Arrival of signal angle is the main aim in DOA. Signal from the line of sight (LoS) at an secondary user, the DoA is represented as:

$$\theta_n = \arctan \frac{\Delta y_k}{\Delta x_k} \tag{3}$$

Where $\Delta x_k = x_p - x_n$, and$\Delta y_k = y_p - y_n$.Angle of arrivals (AoAs) is also assumed as DOA sometimes. The literature study claims various methods for DOA estimation most popular approaches are MUSIC [11] and ESPRIT [12] are based on covariance estimation on received signal. The RIMAX algorithm [13] uses ML algorithm for DOA estimation.

The estimated DoA is commonly modelled as $\widehat{\theta_n} \doteq \theta_n + v_n$ [10], where $v_n \sim N(0, \sigma_n^2)$ and $\sigma_n^2$is the DoA estimation error variance. We denote $\hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \dots \hat{\theta}_N]$ as collections of DoA measurements of all CRs at the fusion centre.

We consider two different modelling of the DoA estimation error variance, using CRB., the result for Uniform Linear Array (ULA) is given by:

$$\sigma_{n\ CRB}^2 = \frac{1}{(k\cos\tilde{\theta}_n)^2} \frac{6}{N_s N_a (N_a^2 - 1)\rho_n} \tag{4}$$

Where $\kappa$ is a constant calculated by the signal wavelength and array spacing, $N_s$ is the number of samples, $N_a$ is the number of antennas, $\tilde{\theta}_n$ is the angular array orientation according to incomingDoA defined as $\tilde{\theta}_n = \theta_n - \bar{\theta}_n$. Where $\bar{\theta}_n$ is orientation of $n^{th}$ ULA. And $\rho_n$ is the (SNR) ratio given by $\rho_n = \frac{\psi_n}{P_M}$, where $P_M$ is noise power.

According to SNR definition we can simplify (4) as

$$\sigma_{n\ ,CRB}^2 = \frac{6P_M}{k^2 N_s N_a (N_a^2 - 1)} \frac{1}{\hat{\psi}_n} \frac{1}{\cos^2 \tilde{\theta}_n}$$

International Journal of Electrical and Electronics Engineers
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$$= \beta f_{CRB}(\hat{\phi}_n)\frac{1}{\cos^2\tilde{\theta}_n}. \quad (5)$$

Where $\beta \triangleq \frac{6F_M}{k^2 N_s N_a (N_a^2-1)}$ and $f_{CRB}(\hat{\phi}_n) \triangleq \frac{1}{\hat{\psi}_n}$. The estimation error variance using ULA is given by

$$\sigma^2_{n,MU} = \frac{1}{(k\cos\tilde{\theta}_n)^2}\frac{6}{N_s N_a (N_a^2-1)\rho_n}\left(1+\frac{1}{N_a\rho_n}\right)$$

$$= \beta f_{MU}(\hat{\phi}_n)\frac{1}{\cos^2\tilde{\theta}_n}. \quad (6)$$

Where $f_{MU}(\hat{\phi}_n) \triangleq \frac{\hat{\psi}_n+(F_M/N_a)}{\hat{\psi}_n^2}$. Note that both (5) and (6) depend on RSS and Primary user Location (used in calculation of $\tilde{\theta}_n$).

PU location estimate $\hat{l_p} \triangleq [\hat{x}_p, \hat{y}_p]^T$ according to RSS and DOA. The Root-Mean-Square-Error (RMSE) of the location estimates is given by:

$$RMSE \triangleq \sqrt{\mathrm{E}\left[\|\hat{l}_p - l_p\|^2\right]} \quad (7)$$

### 2.2 Cramer-Rao Bounds for Fixed Cr Placement

This segment focus more on the concepts of Joint CRB and Finding of RMSE in a static CR structure. forDoA estimates achieved by optimal estimator. Further CRB and RMSE is developed for RSS only localization consequences. By RSS and DOA, the covariance matrix of unbiased estimation of PU locations $\hat{l}_p$ is lower-bounded by means of the CRB.

$$\Omega_{\hat{l}_p} = \triangleq \mathrm{E}\left[(\hat{l}_p - \mathrm{E}[\hat{l}_p])(\hat{l}_p - \mathrm{E}[\hat{l}_p])^T\right] \geq F^{-1} \quad (8)$$

Where $F$ is Fisher Information Matrix (FIM) of size the 2×2, expressed as:

$$F = -\mathrm{E}_{\hat{\theta}\hat{\sigma}}\left[\frac{\partial^2}{\partial l_p^2}\log p(\hat{\theta},\hat{\sigma}|l_p)\right] \quad (9)$$

Consequently the RMSE is limited by RMSE$\geq \sqrt{\{F^{-1}\}_{11}+\{F^{-1}\}_{22}}$, where $\{X\}_{ij}$ represents the $ij^{th}$ component of matrix $X$. Solving conditional probability $p(\hat{\theta},\hat{\sigma}|l_p) = p(\hat{\theta}|\hat{\sigma},l_p)p(\hat{\sigma}|l_p)$, the FIM is decomposed as:

$$F = \left\{-\mathrm{E}_{\hat{\theta}\hat{\sigma}}\left[\frac{\partial^2}{\partial l_p^2}\log p(\hat{\theta}|\hat{\sigma},l_p)\right]\right\} + \left\{-\mathrm{E}_{\hat{\sigma}}\left[\frac{\partial^2}{\partial l_p^2}\log p(\hat{\sigma}|l_p)\right]\right\} \triangleq F_{\hat{\theta}|\hat{\sigma}} + F_{\hat{\sigma}} \quad (10)$$

Note that $F_{\hat{\sigma}}$ is the FIM according to RSS only for primary user localization. The rest part of paper follows the derivation of RSS-only FIM $F_{\hat{\sigma}}$ and further joint FIM $F$ by deriving $F_{\hat{\theta}|\hat{\sigma}}$ for optimal DoA estimator.

### 2.3 RSS-only CRB

To derive the RSS-only FIM $F_{\hat{\sigma}}$, we initially explicitly express the logarithm of the PDF of $\hat{\phi}$ i.e.:

$$\log p(\hat{\sigma}|l_p) = -\log\left[(2\pi)^{\frac{N}{2}}(\det\Omega_s)^{\frac{1}{2}}\right] - \frac{1}{2}(\hat{\phi}-\bar{\phi})^T\Omega_s^{-1}(\hat{\phi}-\bar{\phi}) \quad (11)$$

The RSS-only FIM $F_{\hat{\sigma}}$ is then expressed as:

$$F_{\hat{\sigma}} = \frac{1}{2}\mathrm{E}_{\hat{\sigma}}\left[\frac{\partial^2}{\partial l_p^2}(\hat{\phi}-\bar{\phi})^T\Omega_s^{-1}(\hat{\phi}-\bar{\phi})\right] \quad (12)$$

International Journal of Electrical and Electronics Engineers
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ISSN (O) 2321-2055
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The elements of $F_{\hat{\delta}}$ are expressed as:

$$\{F_{\hat{\delta}}\}_{11} = \frac{\partial(\hat{\phi}-\bar{\phi})^T}{\partial x_p}\Omega_s^{-1}\frac{\partial(\hat{\phi}-\bar{\phi})}{\partial x_p}$$

$$= \epsilon\gamma^2\Delta x^T D^{-2}\Omega_s^{-1}D^{-2}\Delta x$$

$$\{F_{\hat{\delta}}\}_{22} = \frac{\partial(\hat{\phi}-\bar{\phi})^T}{\partial y_p}\Omega_s^{-1}\frac{\partial(\hat{\phi}-\bar{\phi})}{\partial y_p}$$

$$= \epsilon\gamma^2\Delta y^T D^{-2}\Omega_s^{-1}D^{-2}\Delta y$$

$$\{F_{\hat{\delta}}\}_{12} = \{F_{\hat{\delta}}\}_{21} = \frac{\partial(\hat{\phi}-\bar{\phi})^T}{\partial x_p}\Omega_s^{-1}\frac{\partial(\hat{\phi}-\bar{\phi})}{\partial y_p}$$

$$= \epsilon\gamma^2\Delta x^T D^{-2}\Omega_s^{-1}D^{-2}\Delta y \qquad (13)$$

Where $\epsilon = 100/(\log 10)^2$, and vectors and matrices are defined as:

$D \triangleq \mathrm{diag}(d_1, d_2 \dots, d_N)$,

$\Delta x \triangleq [\Delta x_1, \Delta x_2, \dots, \Delta x_N]^T$,

$\Delta y \triangleq [\Delta y_1, \Delta y_2, \dots, \Delta y_N]^T$,

$\Delta x_N = x_p - x_n$and

$\Delta y_N = y_p - y_n$

To obtain a compact expression of $F_{\hat{\delta}}$, let us define $L = [\Delta x, \Delta y]^T$ and $\Lambda = \frac{1}{\epsilon\gamma^2}D^2\Omega_s D^2$. Hence, it is clearly verify that the FIM and RMSE of RSS only PU localization are expressed as:

$$F_{\hat{\delta}} = L\Lambda^{-1}L^T \qquad (14)$$

$$RMSE_{R,F} \geq \sqrt{\{F_{\hat{\delta}}^{-1}\}_{11} + \{F_{\hat{\delta}}^{-1}F_{\hat{\delta}}^{-1}\}_{22}} \qquad (15)$$

Where the subscript $R, F$ represents RSS-only bound for fixed placement.

### 2.4Joint CRB using Optimal DoA Estimator

This heading derive the joint CRB with DoA estimations given by the optimal estimator, using the DoA error variance given by $\sigma^2_{n,CRB}$. To derive the conditional FIM of DoA given RSS $F_{\hat{\theta}|\hat{\delta}}$, we first explicitly express logarithm of the conditional PDF $p(\hat{\theta}|\hat{\emptyset}, l_p)$ as:

$$\log p(\hat{\theta}|\hat{\emptyset}, l_p) = \log\left[\prod_{n=1}^{N}\frac{1}{\sqrt{2\pi\sigma^2_{n,CRB}}}\exp\left\{-\frac{(\hat{\theta}_n-\theta_n)^2}{2\sigma^2_{n,CRB}}\right\}\right]$$

$$= \sum_{n=1}^{N}\left[\log(\cos\hat{\theta}_n) - \frac{1}{2}\log\{2\pi\beta f_{CRB}(\hat{\emptyset}_n)\} - \frac{\cos^2\hat{\theta}_n(\hat{\theta}_n-\theta_n)^2}{2\beta f_{CRB}(\hat{\emptyset}_n)}\right] \qquad (16)$$

Then $F_{\hat{\theta}|\hat{\delta}}$ is expressed as:

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$$F_{\tilde{\theta}|\hat{\delta}} = \sum_{n=1}^{N}\left[\mathbb{E}_{\tilde{\theta},\hat{\delta}}\left\{\frac{\partial^2 g_n}{\partial l_p^2}\right\} - \mathbb{E}_{\tilde{\theta},\hat{\delta}}\left\{\frac{\partial^2 h_n}{\partial l_p^2}\right\}\right] \quad (17)$$

Where, $g_n \triangleq \frac{\cos^2\tilde{\theta}_n(\hat{\theta}_n-\theta_n)^2}{2\beta f_{CRB}(\tilde{\theta}_n)}$ and $h_n \triangleq \log(\cos\tilde{\theta}_n)$. The elements of $F_{\tilde{\theta}|\hat{\delta}}$ are derived as:

$$\{F_{\tilde{\theta}|\hat{\delta}}\}_{11} = \sum_{n=1}^{N}\frac{\Delta y_n^2}{d_n^4}\left\{\frac{\alpha\cos^2\hat{\theta}_n}{d_n^{\gamma}} + 2\tan^2\hat{\theta}_n\right\}$$

$$\{F_{\tilde{\theta}|\hat{\delta}}\}_{22} = \sum_{n=1}^{N}\frac{\Delta x_n^2}{d_n^4}\left\{\frac{\alpha\cos^2\hat{\theta}_n}{d_n^{\gamma}} + 2\tan^2\hat{\theta}_n\right\}$$

$$\{F_{\tilde{\theta}|\hat{\delta}}\}_{12} = \{F_{\tilde{\theta}|\hat{\delta}}\}_{21} = \sum_{n=1}^{N}\frac{\Delta x_n \Delta y_n}{d_n^4}\left\{\frac{\alpha\cos^2\hat{\theta}_n}{d_n^{\gamma}} + 2\tan^2\hat{\theta}_n\right\} \quad (18)$$

Where, $\alpha \triangleq c_0 P_T e^{\sigma_s^2/(2\epsilon)}/\beta$.

To obtain a compact expression of $F_{\tilde{\theta}|\hat{\delta}}$, let us define $P = [\Delta y - \Delta x]^T$ and $\Gamma = \mathrm{diag}(\gamma_1, \gamma_2, \ldots, \gamma_n)$, where $\gamma_n = \frac{1}{d_n^4}\left\{\frac{\alpha\cos^2\hat{\theta}_n}{d_n^{\gamma}} + 2\tan^2\hat{\theta}_n\right\}$. Hence, it is clearly verify that the $F_{\tilde{\theta}|\hat{\delta}} = P\Gamma P^T$ . Consequently, the joint FIM and the equivalent RMSE are expressed as:

$$F_{J,F,C} = P\Gamma P^T + L\Lambda^{-1}L^T \quad (19)$$

$$RMSE_{J,F,C} \geq \sqrt{\{F_{J,F,C}^{-1}\}_{11} + \{F_{J,F,C}^{-1}\}_{22}} \quad (20)$$

Where the subscript $J,F,C$ represents joint CRB for fixed placement using CRB of DoA estimation error variance.

## III.SIMULATION AND RESULTS

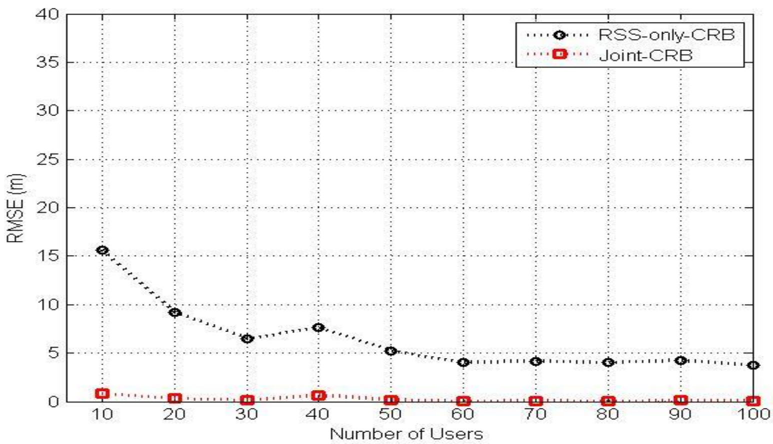


**Figure.2: Results for RMSE of RSS-only CRB and Joint CRBwith varying number of CRs**

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## IV. CONCLUSION:

The joint CRB and the relating bound on RMSE for a static CR arrangement, for DoA estimates acquired from optimal estimator. We additionally develop the CRB and RMSE for RSS only localization consequence. Utilizing RSS and DoA as estimations, the covariance matrix of unbiased estimation of PU locations $\hat{l}_p$ is lower-bounded by means of the CRB. Hence

Observation found that joint RSS-DOA outperformed then RSS only for different user.

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