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PERFORMANCE ANALYSIS OF SPACE DIVERSITY IN a-μ FADING CHANNEL

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ABSTRACT

The α-μ distribution is a generalized fading distribution which explores nonlinearities of the wireless channel. The performance metrics such as outage and bit error rate (BER) of α-μ distribution for maximal ratio combining (MRC), equal gain combining (EGC), selection combining (SC) and switch and stay combining (SSC) is evaluated analytically and by simulation. Analytical and simulation results are discussed in this paper.

Keywords: Multipath Propagation, Diversity Combining, A-μ Distribution, Bit Error Rate, Outage Probability.

I. INTRODUCTION

In the recent past alpha-mu (α-μ) fading model [1] has been proposed to describe the mobile radio signal considering two important phenomenon of radio propagation non-linearity and clustering. The α-μ represents a generalized fading distribution for small-scale variation of the fading signal in a non line-of-sight fading condition. The α-μ distribution is flexible, general and has easy mathematical tractability. In fact, the Generalized Gamma Distribution (GGD) also known as Stacy distribution [2] has been renamed as α-μ distribution. As given in its name, alpha-mu distribution is written in terms of two physical parameters, namely α and μ. The power parameter ($\alpha > 0$) is related to the non-linearity of the environment i.e. propagation medium, whereas the parameter ($\mu > 0$) is associated to the number of multipath clusters.

In [1,2] the probability density function has been obtained. The accurate approximations for the outage probability of equal gain receivers subject to arbitrary independent co-channel interferers are proposed in [3]. Based on moment estimators approach closed form approximations for the level crossing rate (LCR) of multi-branch equal-gain and maximal-ratio combiners operating on independent non-identically distributed Nakagami-m fading channels has been derived in [4]. The exact expressions for LCR and average fade duration (AFD) of equal-gain and maximal-ratio combiners have been derived in [5]. Moment generating function (MGF) for the PDF characterizing of α-μ fading channel has been derived in [6]. MGF is further used for evaluating bit error rate. Reference [7] derive the switching rate of a dual branch selection diversity combiner for generalized fading. Performance analysis of dual selection combining diversity receiver over correlated α-μ fading channels is presented in [8]. Reference [9] provides performance analysis of signal-to-interference ratio based selection combining diversity system over α-μ fading distributed and correlated channels. Combining techniques like EGC, MRC require all or some of the amount of the channel state information of received signal. Whereas SC receiver processes only one of the diversity branches. The performance of a dual-branch switched-and-stay combining diversity receiver, operating over correlated α-μ fading channel is discussed in [10]. The performance of the system with dual selection combining over correlated Weibull channel in the presence of α-μ distributed co-channel interference is studied in [11]. Bit error rate for i.i.d. α-μ fading channel with a maximal ratio combining receiver is carried out in [12].

The paper is organized as follows. In Sections 2, we revisit the alpha-mu fading model. Probability density function, signal to noise ratio, outage and bit error rate are evaluated. In Section 3, short review of the various diversity schemes such as MRC, EGC, EC and SSC is presented. Monte-carlo simulation results are presented in Section 4. The paper is concluded by Section 5.

II. THE ALPHA-MU FADING MODEL REVISITED

In the α-μ distribution, it is considered that a signal is composed of clusters of multipath waves propagating in a non-homogenous environment. In any one of the cluster, the phases of the scattered waves are random and have similar delay times. Further, the delay-time spreads of different clusters is generally relatively large. It is assumed that the clusters of multipath waves have the scattered waves with identical powers. As a result, the obtained envelope, is a non-linear function of the modulus of the sum of the multipath components.

Assuming that the received signal at the i^{th} branch (i =1, ...,M) includes a certain number n_i of multipath clusters, the resulting α-μ envelope R_i at the i^{th} branch is written as

$$R_i^{\alpha_i} = \sum_{i=1}^{n_i}\left(X_{il}^2 + Y_{il}^2\right) \qquad (1)$$

where $\alpha_i > 0$ is the power parameter, X_{il} and Y_{il} are zero mean mutually independent Gaussian processes with identical variances i.e.

$$V(X_{il}) = V(Y_{il}) = \sigma_i^2 = \frac{\hat{r}_i^{\alpha_i}}{2n_i}$$

$$\hat{r}_i = \text{the } \alpha_{\text{i}} - \text{root mean value of } R_i^{\alpha_i} = \sqrt[\alpha_i]{E(R_i^{\alpha_i})}$$

with E(·) and V (·) denoting mean and variance operators respectively. Thus, for a (α-μ) fading signal with envelope R, an arbitrary parameter ($\alpha > 0$), and a α-root mean value of R^α is given as

$$\hat{r} = \sqrt[\alpha]{E(R^\alpha)} = \sqrt[\alpha]{2\mu\sigma^2}$$

2.1 Probability Density Function of α-μ

The α-μ probability density function, $f_R(r)$ of R is given [1] as

$$f_R(r) = \frac{\alpha\ \mu^\mu\ \mathrm{r}^{\alpha\mu - 1}}{\hat{r}^{\alpha\mu}\ \Gamma(\mu)} \quad \exp\left[-\mu\frac{r^\alpha}{\hat{r}^\alpha}\right] \qquad (2)$$

where $\mu \geq 0$, is the inverse of the normalized variance of R^α, given as,

$$\mu = \frac{\mathrm{E}^2(R^\alpha)}{V(R^\alpha)}$$

$$\Gamma(z)=\int_0^\infty t^{z-1}\, e^{(-t)}\, dt \quad \text{is the Gamma function.}$$

2.2 Signal to Noise Ratio (SNR) of α-µ

$$\text{Instantaneous SNR, } \gamma=\bar{\gamma}\left(\frac{r}{\hat{r}}\right)^2 \ ; \text{ where } \bar{\gamma}=E\left[\hat{r}^2\right]\frac{E_b}{N_o}$$

and E_b / N_o is the energy per bit to the noise power spectral density ratio. PDF of SNR, $f_\gamma(\gamma)$ can be found by random variable transformation on (2). Let X be a RV, with PDF $f(x)$, and also $X = v(Y)$, then by change of variable tecl $f_Y(y) = f_X(v(y)) \times v'(y)$

On solving, we get PDF of SNR

$$f_\gamma(\gamma)=\frac{\alpha\,\mu^\mu\left[\hat{r}\left\{\frac{\gamma}{\bar{\gamma}}\right\}^{\frac{1}{2}}\right]^{\alpha\mu-1}}{\hat{r}^{\alpha\mu}\,\Gamma(\mu)}\ \exp\left[-\mu\frac{\left\{\hat{r}\left(\frac{\gamma}{\bar{\gamma}}\right)^{1/2}\right\}^{\alpha}}{\hat{r}^{\alpha}}\right]\times\frac{\hat{r}}{2\left(\gamma\bar{\gamma}\right)^{\frac{1}{2}}}$$

$$f_\gamma(\gamma)=\frac{\alpha\,\mu^\mu\,\gamma^{\frac{\alpha\mu}{2}-1}}{2\,\Gamma(\mu)\,\bar{\gamma}^{\alpha\mu/2}}\ e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} \tag{3}$$

2.3 Outage Probability (P_{out}) of α-µ

The outage probability is defined as the probability that the error rate exceeds a pre-defined value or equivalently, the received SNR drops below a pre-defined threshold (γ_{thr}).

$$P_{out}=\int_0^{\gamma_{thr}} f_\gamma(\gamma)\,d\gamma$$

$$P_{out}=\frac{\alpha\,\mu^\mu}{2\,\Gamma(\mu)\,\bar{\gamma}^{\alpha\mu/2}}\int_0^{\gamma_{thr}}\gamma^{\frac{\alpha\mu}{2}-1}e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}}\,d\gamma$$

This integration is solved by using equation 3.381.8 of [13] given as

$$\int_0^u x^m e^{-\beta x^n}\,dx=\frac{\gamma(v,\ \beta u^n)}{n\beta^v},\ \text{where } v=\frac{m+1}{n}$$

On comparison we find that

$$u=\gamma_{thr},\quad x=v,\quad m=\frac{\alpha\mu}{2}-1,\quad \beta=\frac{\mu}{\bar{\gamma}^{\alpha/2}},\quad n=\alpha/2,\quad v=\frac{m+1}{n}=\frac{\frac{\alpha\mu}{2}-1+1}{\alpha/2}=\mu$$

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Therefore

$$P_{out} = \frac{\alpha\,\mu^{\mu}}{2\,\Gamma(\mu)\,\bar{\gamma}^{\alpha\mu/2}} \; \frac{\gamma\left(\dfrac{\frac{\alpha\mu}{2}-1+1}{\alpha/2},\; \dfrac{\mu}{\bar{\gamma}^{\alpha/2}}\,\gamma_{thr}^{\alpha/2}\right)}{\dfrac{\alpha}{2}\left(\dfrac{\mu}{\bar{\gamma}^{\alpha/2}}\right)^{\mu}}$$

$$P_{out} = \frac{\gamma_{thr}\left(\mu,\; \mu\left(\dfrac{\gamma_{thr}}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(u)} \qquad (4)$$

This is also the expression for cumulative distribution function (CDF) of α-μ fading distribution.

2.4 Bit Error Rate of α-μ

The expression of BER for α-μ fading channel obtained in [6] through MGF approach is

$$P_e(\psi) = \frac{\alpha\mu^{\mu}}{\Gamma(\mu)\,\bar{\gamma}^{\alpha\mu/2}} \, \frac{\mathrm{k}^{\frac{1}{2}}\, l^{\frac{\alpha\mu-1}{2}}}{(2\pi)^{\frac{l+k}{2}}} \, I_{GI}(\psi) \qquad (5)$$

$$\text{Where,}\quad I_{GI}(\psi) = \frac{l^{-\frac{1}{2}}\,\Gamma^{2}\left(\frac{1}{2}\right)}{2\psi^{\alpha\mu/2}}$$

$$\times G_{2l,k+l}^{k,2l}\left(\omega \,\middle|\, \begin{matrix} I\left(l,\dfrac{1-\alpha\mu}{2}\right), & I\left(l,1-\dfrac{\alpha\mu}{2}\right) \\ I(k,0), & I\left(l,-\dfrac{\alpha\mu}{2}\right) \end{matrix}\right)$$

III. DIVERSITY COMBINING TECHNIQUES

With diversity technique multiple copies of the same signal is received on different branches, which undergo independent fading. If one branch undergoes a deep fade, the another branch may have strong signal. In space diversity fading are minimized by the simultaneous use of two or more physically separated antennas. Thus having more than one path to select the SNR at receiver may be improved by selecting appropriate combining technique. The SNR γ is a random variable and is given for different diversity schemes.

$$\gamma \;\in\; \{\gamma_{SC},\ \gamma_{MRC},\ \gamma_{EGC},\ \gamma_{SSC}\}$$

Following diversity combining techniques for α-μ fading distribution are discussed.

3.1 Selection Combining (SC)

Selection combining is based on the principle of selecting the best signal among all the signals received from different branches at the receiving end. In this method, the receiver monitors the SNR of the incoming signal using switch logic. The branch with highest instantaneous SNR is connected to demodulator.

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SNR of selection combining is given as

$$\gamma_{SC} = \text{Max}\left(R_1^2, R_2^2\right) \times SNR \qquad (6)$$

Where R_1 and R_2 represent the fading envelope for two channels seen by two different antennas. R_1^2 is fading power between transmitter and 1^{st} receiving antenna. R_2^2 is fading power between transmitter and 2^{nd} receiving antenna. *SNR* is unfaded signal to noise ratio. PDF of selection combining SNR (γ) can be obtained by differentiating the CDF of the α-μ fading distribution obtained in (4) above.

$$f_{\gamma_{SC}}(\gamma) = \frac{d}{d\gamma}\left[\frac{\gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(u)}\right]^L \qquad (7)$$

Using definition of $\gamma(s, x)$ i.e. Lower Incomplete Gamma function as, $\gamma(s,x) = \int_0^x t^{s-1} e^{-t}\, dt$

We get, $$f_{\gamma_{SC}}(\gamma) = L\left[\frac{\gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(u)}\right]^{L-1} \frac{d}{d\gamma}\left[\frac{\int_0^{\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} t^{\mu-1} e^{-t}\, dt}{\Gamma(u)}\right]$$

On solving, the PDF of selection combining over α-μ fading channel is obtained as

$$f_{\gamma_{SC}}(\gamma) = \frac{L\, \alpha\, \mu^{\mu}\, \gamma^{\frac{\alpha\mu}{2}-1}}{2\,\Gamma(u)\, \bar{\gamma}^{\frac{\alpha\mu}{2}}}\, e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} \left[\frac{\gamma\left(\mu, \mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right)}{\Gamma(u)}\right]^{L-1} \qquad (8)$$

The probability for both channels (L=2) receiving signal which are also less than the threshold γ is

$$\Pr[\gamma_1, \gamma_2 \leq \gamma] = P_M(\gamma) \qquad (9)$$

Where, $\gamma_1 = R_1^2, \;\; \gamma_2 = R_2^2$

3.2 Maximal Ratio Combining (MRC)

This is the most complex scheme in which all branches are optimally combined at the receiver. MRC requires scaling and co-phasing of individual branch. In this all the signals are weighted according to their individual signal to noise power ratios and then summed. Thus MRC produces an output SNR, which is equal to the sum of the individual SNRs. The advantage of MRC is producing an output with an acceptable SNR even when none of the individual signals are themselves acceptable. Best statistical reduction of fading is achieved by this technique. SNR (γ_{MRC}) of MRC for α-μ fading distribution is given as

$$\gamma_{MRC} = \left(R_1^2 + R_2^2\right) \times SNR \qquad (10)$$

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3.3 Equal Gain Combining (EGC)

A variant of MRC is equal gain combining (EGC) [6], where signals from each branch are co-phased and their weights have equal magnitude. This method also has possibility like MRC of producing an acceptable output signal from a number of unacceptable input signals. SNR improvement of EGC is better than selection combining but not better than MRC. SNR of EGC (γ_{EGC}) is given as

$$\gamma_{EGC} = \frac{1}{2}\left(R_1+R_2\right)^2 \times SNR \qquad (11)$$

3.4 Switch & Stay Combining (SSC)

SSC further simplifies the complexities of SC. In this in place of continually connecting the diversity path with best quality, a particular diversity path is selected by the receiver till the quality of the path drops below a predetermined threshold. When it happens, then the receiver switches to another diversity path. This reduces the complexities relative to SC, because continuous and simultaneous monitoring of all diversity path in not required. The CDF of SNR of SSC is given [14] as

$$F_{\gamma_{ssc}} = \begin{cases} \Pr\left[\left(\gamma_1 \le \gamma_T\right) and \left(\gamma_2 \le \gamma\right)\right], & \gamma < \gamma_T \\ \Pr\left[\left(\gamma_T \le \gamma_1 \le \gamma\right) or \left(\gamma_1 \le \gamma_T and \gamma_2 \le \gamma\right)\right], & \gamma \ge \gamma_T \end{cases}$$

This can be expressed in terms of CDF of individual branches

$$F_{\gamma_{ssc}} = \begin{cases} F_\gamma\left(\gamma_T\right) F_\gamma\left(\gamma\right), & \gamma < \gamma_T \\ F_\gamma\left(\gamma\right) - F_\gamma\left(\gamma_T\right) + F_\gamma\left(\gamma\right) F_\gamma\left(\gamma_T\right), & \gamma \ge \gamma_T \end{cases}$$

PDF is obtained by differentiating CDF as below

$$f_{\gamma_{ssc}} = \frac{\mathrm{d}\, F_{\gamma SSC}\left(\gamma\right)}{\mathrm{d}\gamma} = \begin{cases} F_\gamma\left(\gamma_T\right) f_\gamma\left(\gamma\right), & \gamma < \gamma_T \\ \left(1+F_\gamma\left(\gamma_T\right)\right) f_\gamma\left(\gamma\right), & \gamma \ge \gamma_T \end{cases} \qquad (12)$$

IV. SIMULATION RESULTS AND DISCUSSIONS

Outage performance and BER for different combining techniques for α-μ fading channel is obtained by Monte-Carlo simulation are shown in Fig. 1 to Fig.4. In these simulation 1000000 samples have been considered for a particular α and μ combination.

In Fig.1 and Fig.2 outage performance and BER are shown for α=3, μ=1. It is observed that performance of MRC is better among all and subsequent performance is of EGC, SC and SSC. In Fig.3 and Fig.4 outage performance and BER for α=3, μ=3 are shown. It is observed that performance of MRC is better among all and then the EGC, SC and SSC follow the performance subsequently. The MRC is optimal combining scheme but it is at the expense of complexity. MRC requires knowledge of channel amplitude and phase, hence can be used for M-QAM or for any amplitude/phase modulations. On the other hand the SC combiner chooses the branch with highest SNR i.e. output is equal to the signal on only one of the branches, hence it does not require knowledge of the signal phases on each branch as in the case of MRC or EGC. The conventional SC is impractical because it requires the simultaneous and continuous monitoring of all the diversity branches.

Therefore the SC is implemented in switched form i.e. SSC, where in place of continuously picking the best branch, receiver remains on a particular branch till its SNR drops below a specified threshold (γ_T=0.75). That is why SSC performance is slightly poor than SC.

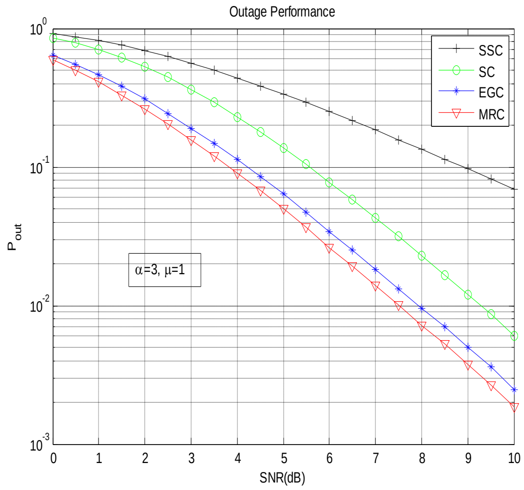

Fig. 1. Outage of α-μ fading channel for α=3 and μ=1

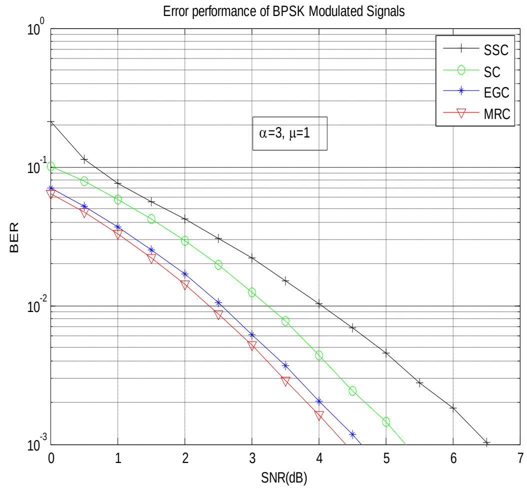

Fig. 2. BER of α-μ fading channel for α=3 and μ=1

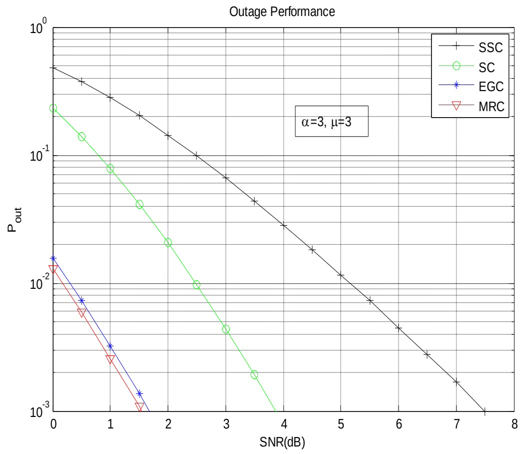

Fig. 3. Outage of α-μ fading channel for α=3 and μ=3

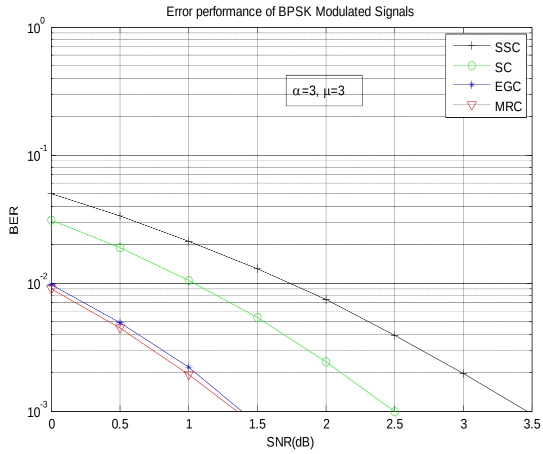

Fig. 4. BER of α-μ fading channel for α=3 and μ=3

Further, it is also seen that by increasing μ from 1 to 3, the improvement in outage and BER is observed. Increase in μ, indicates increase in clusters. Thus, inference can be drawn that by increasing the clusters outage and BER improves.

IV. CONCLUSION

In this paper PDF, CDF, SNR, outage and BER of α-μ fading model have been briefly discussed. The simulated and analytical results of performance metrics such as outage and BER for MRC, EGC, SC and SSC spatial diversity combining schemes have been illustrated. The effect of α and μ parameters variation on BER and outage is brought out. Comparison between different diversity schemes have been discussed with help of simulation results. The result obtained in this letter motivates researcher to explore more the α-μ generalized fading model.

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